

20. 1-3 三个数随机抽1个

A: 抽出1或2

B: 抽出2

C: 抽出2/3

$$P(A) = \frac{2}{3}$$

$$P(A|B) = 1 \quad P(A|B) > P(A)$$

$$P(B) = \frac{1}{3} \quad P(B|C) > P(B)$$

$$P(B|C) = \frac{1}{2}$$

$$\text{但 } P(A|C) = \frac{1}{2} \quad P(A|C) < P(A)$$

$$23. (1) \quad P_1 = \frac{n \times (n-1)^{r-1}}{n^r} = \left(\frac{n-1}{n}\right)^{r-1}$$

$$(2) \quad P_2 = \begin{cases} \frac{n \times (n-1) \times \dots \times (n-r+1)}{n^r}, & r \leq n \\ 0, & r > n \end{cases}$$

$$(3) \quad P_3 = P_1^m \quad (\text{独立})$$

$$P_4 = \begin{cases} \frac{n(n-1) \dots (n-rm+1)}{n^{rm}}, & rm \leq n \\ 0, & rm > n \end{cases}$$

24. (1) 设 A: 取一等品. B_1 : 取第一箱. B_2 : 取第二箱

$$P(A) = P(A|B_1)P(B_1) + P(A|B_2)P(B_2)$$

$$\frac{1}{5} \times \frac{1}{2} + \frac{3}{5} \times \frac{1}{2} = \frac{2}{5}$$

(2) 设 C: 第二次也取一等品

$$\begin{aligned} P(C|A) &= \frac{P(AC)}{P(A)} = \frac{P(A|B_1)P(B_1) + P(A|B_2)P(B_2)}{P(A)} \\ &= \frac{\frac{1}{2} \times \frac{1}{5} \times \frac{9}{49} + \frac{1}{2} \times \frac{3}{5} \times \frac{17}{49}}{\frac{2}{5}} \\ &= 48.56\% \end{aligned}$$

30.(1) A: 阳性 B: 带菌

$$\begin{aligned} P(B|A) &= \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|\bar{B})P(\bar{B})} \\ &= \frac{0.95 \times 0.1}{0.95 \times 0.1 + 0.01 \times 0.9} \\ &= 91.3\% \end{aligned}$$

(2) C: 两次阳

$$\begin{aligned} P(B|C) &= \frac{P(C|B)P(B)}{P(C|B)P(B) + P(C|\bar{B})P(\bar{B})} = \frac{0.95^2 \times 0.1}{0.95^2 \times 0.1 + 0.01^2 \times 0.9} \\ &= 99.9\% \end{aligned}$$

33. A_i : 甲中放了 i 个白球到乙

B : 从乙摸白

$$P(A_0) = \frac{C_2^2}{C_7^2} = \frac{1}{21}$$

$$P(B|A_0) = \frac{4}{11}$$

$$P(A_1) = \frac{C_5^1 C_2^1}{C_7^2} = \frac{10}{21}$$

$$P(B|A_1) = \frac{5}{11}$$

$$P(A_2) = \frac{C_5^2}{C_7^2} = \frac{10}{21}$$

$$P(B|A_2) = \frac{6}{11}$$

$$(1) P(B) = \sum_{i=0}^2 P(A_i) P(B|A_i) = \frac{38}{77} \approx 49.36\%$$

$$(2) P = 1 - P(A_0|B) = 1 - \frac{P(B|A_0)P(A_0)}{P(B)} = 1 - \frac{\frac{1}{21} \times \frac{4}{11}}{\frac{38}{77}} = \frac{37}{38} \approx 97.37\%$$